ECE 313: Electromagnetic Waves

Lecture 6: fields in free space

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Electromagnetic wave equation

$$\nabla \times \overline{E} = -\frac{\partial B}{\partial t}$$

$$\nabla \times \nabla \times \overline{E} = -\mu \frac{\partial (\nabla \times H)}{\partial t} = -\mu \frac{\partial}{\partial t}(\overline{J} + \varepsilon \frac{\partial \overline{E}}{\partial t}), \quad \overline{J} = \sigma \overline{E}$$

$$\nabla(\nabla . \overline{E}) - \nabla^2 \overline{E} = -\mu\sigma \frac{\partial \overline{E}}{\partial t} - \mu\varepsilon \frac{\partial^2 \overline{E}}{\partial t^2}$$

$$\nabla(\frac{\rho}{\varepsilon}) - \nabla^2 \overline{E} = -\mu\sigma \frac{\partial \overline{E}}{\partial t} - \mu\varepsilon \frac{\partial^2 \overline{E}}{\partial t^2}$$

$$\nabla^2 \overline{E} - \mu\sigma \frac{\partial \overline{E}}{\partial t} - \mu\varepsilon \frac{\partial^2 \overline{E}}{\partial t^2} = \nabla(\frac{\rho}{\varepsilon})$$

General form of wave equation for electric field

$$if \;\; source \;\; free \;\; \rho = 0, \;\; free \; space \; \sigma = 0 \;\; \mu = \mu_0 \quad \varepsilon = \varepsilon_0$$

$$\nabla^2 \overline{E} - \mu_0 \varepsilon_0 \frac{\partial^2 \overline{E}}{\partial t^2} = 0$$

similarly

$$\nabla^2 \mathbf{H} - \frac{1}{u^2} \frac{\partial^2 \mathbf{H}}{\partial t^2} = 0.$$

Electromagnetic wave equation

$$\nabla^2 \overline{E} - \mu\sigma \frac{\partial \overline{E}}{\partial t} - \mu\varepsilon \frac{\partial^2 \overline{E}}{\partial t^2} = \nabla\left(\frac{\rho}{\varepsilon}\right)$$

$$source\ \ free\ \rho = 0$$

$$\nabla^2 \overline{E} - \mu\sigma \frac{\partial \overline{E}}{\partial t} - \mu\varepsilon \frac{\partial^2 \overline{E}}{\partial t^2} = 0$$

$$assume\ time\ \sin usoidal\ \ \text{var}\, iation\ \ \overline{E}(x, y, z, t) = \overline{E}(x, y, z)e^{j\omega t}$$

$$\nabla^2 \overline{E} - j\omega\mu\sigma\overline{E} + \omega^2 \mu\varepsilon\overline{E} = 0$$

$$\nabla^2 \overline{E} + \gamma^2 \overline{E} = 0 \rightarrow \gamma^2 = -j\omega\mu\sigma + \omega^2 \mu\varepsilon$$

$$\gamma = \sqrt{-j\omega\mu\sigma + \omega^2 \mu\varepsilon} \rightarrow j\gamma = j\sqrt{\omega^2 \mu\varepsilon - j\omega\mu\sigma} = \alpha + j\beta$$

Propagation constant (m^{-1})

Attenuation constant (Neper/m)

Phase shift constant (rad/m)

Wave propagation with free source :

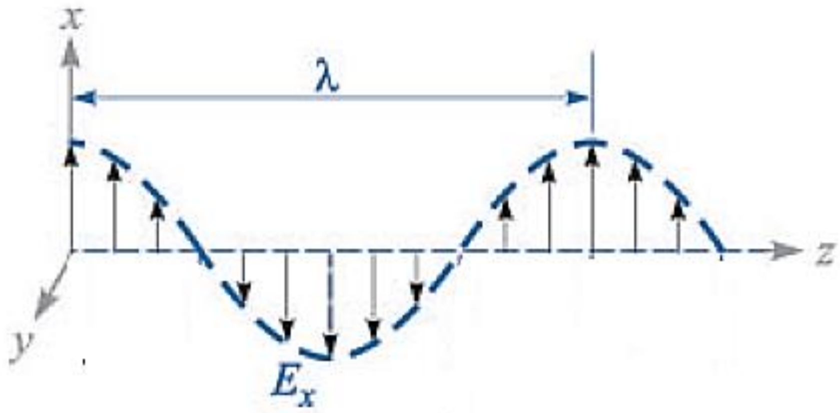
$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$\bar{E} = E_x(z)\hat{x}$$

Let E doe not vary with x or y

$$\frac{\partial^2}{\partial x^2} = 0, \quad \frac{\partial^2}{\partial y^2} = 0$$

$$\frac{\partial^2 E_x(z)}{\partial z^2} + \gamma^2 E_x(z) = 0$$



E is polarized in x direction
Propagation in Z direction $e^{\pm j\beta z}$

$$E_x(z) = Ae^{-j\gamma z} + Be^{+j\gamma z}$$

$$A = E_0^+ e^{j\varphi_1}, \quad B = E_0^- e^{j\varphi_2}$$

$$E_x(z) = E_0^+ e^{-\alpha z} e^{j\varphi_1} e^{-j\beta z} + E_0^- e^{+\alpha z} e^{+j\beta z} e^{j\varphi_2}$$

$$E_x(z,t) = \mathrm{Re}(E_0^+ e^{-\alpha z} e^{j\varphi_1} e^{-j\beta z} e^{j\omega t} + E_0^- e^{+\alpha z} e^{+j\beta z} e^{j\varphi_2} e^{j\omega t})$$

Wave propagate in +z direction *Wave propagate in -z direction*

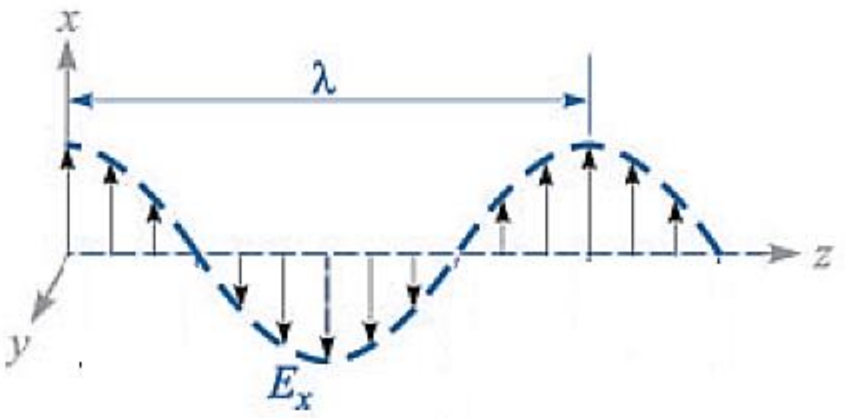
Electromagnetic wave equation

$$E_x(z,t) = E_0^+ e^{-\alpha z}\cos(\omega t - \beta z + \varphi_1) + E_0^- e^{+\alpha z}\cos(\omega t + \beta z + \varphi_2)$$

$$phase\ velocity:$$

$$\omega t_1 - \beta z_1 + \varphi_1 = \omega t_2 - \beta z_2 + \varphi_1 = const$$

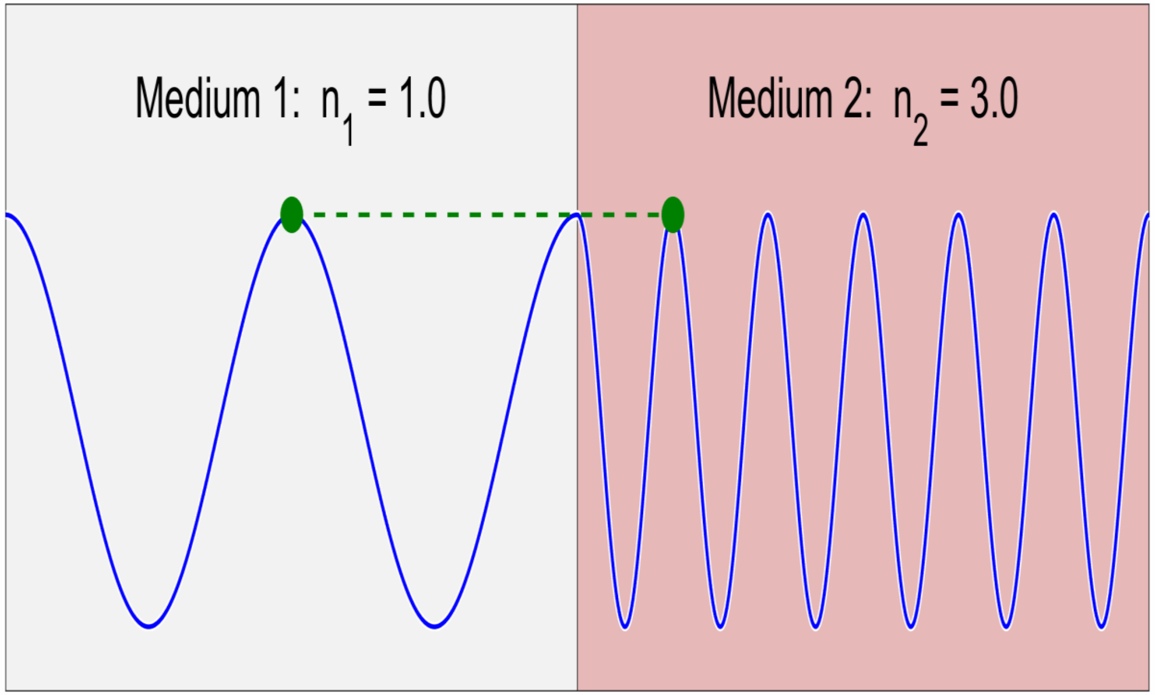
$$v = \frac{\Delta z}{\Delta t} = \frac{\omega}{\beta}$$



$$\beta = \frac{2\pi}{\lambda} = \omega\sqrt{\mu\varepsilon},\quad \omega = 2\pi f = 2\pi\frac{v}{\lambda},\quad v = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{c}{\sqrt{\mu_r\varepsilon_r}}$$

Constant Frequency; Different wavelength and different velocity

When a wave enters a different material, its speed and wavelength changes



Assume no losses(no decaying)

n, μ and ε are pure real

$$\lambda = \frac{\lambda_0}{n} \qquad v = \frac{c}{n} \qquad v_2 < v_1$$

$$n = \sqrt{\mu_r \varepsilon_r} \qquad n \geq 1 \qquad c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

Example 11.3

1 MHz plane wave propagating in fresh water. At this
frequency, losses in water are known to be small, so for simplicity, we will neglect ϵ''. In water, $\mu_R = 1$ and at 1 MHz, $\epsilon'_R = \epsilon_R = 81$.
determine the wavelength , phase velocity and intrinsic impedance

Solution. We begin by calculating the phase constant. Using (36) with $\epsilon'' = 0$, we have

$$\beta = \omega\sqrt{\mu\epsilon'} = \omega\sqrt{\mu_0\epsilon_0}\sqrt{\epsilon'_R} = \frac{\omega\sqrt{\epsilon'_R}}{c} = \frac{2\pi \times 10^6\sqrt{81}}{3.0 \times 10^8} = 0.19 \text{ rad/m}$$

Using this result, we can determine the wavelength and phase velocity:

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{.19} = 33 \text{ m}$$

$$v_p = \frac{\omega}{\beta} = \frac{2\pi \times 10^6}{.19} = 3.3 \times 10^7 \text{ m/s}$$

The wavelength in air would have been 300 m. Continuing our calculations, we find the intrinsic impedance, using (39) with $\epsilon'' = 0$:

$$\eta = \sqrt{\frac{\mu}{\epsilon'}} = \frac{\eta_0}{\sqrt{\epsilon'_R}} = \frac{377}{9} = 42\ \Omega$$

Example:

D11.2 Let $\mathbf{H_s} = (2\angle -40^\circ \mathbf{a_x} - 3\angle 20^\circ \mathbf{a_y})e^{-j0.07z}$ A/m for a uniform plane wave traveling in free space. Find: (a) ω; (b) H_x at $P(1, 2, 3)$ at $t = 31$ ns; (c) $|\mathbf{H}|$ at $t = 0$ at the origin.

$$H_s = 2\cos(\omega t - \beta z + \varphi)\hat{x} + 3\cos(\omega t - \beta z + \varphi')\hat{y}$$

(a)

$$\beta = \omega\sqrt{\mu\varepsilon} \rightarrow \omega = 0.07*3*10^8 = 21*10^6 \, rad/\sec$$

(b)

$$H_s = 2\cos(21*10^6*3*10^{-9} - 0.07*3 - \frac{40}{180}\pi)\hat{x} + 3\cos(21*10^6*3*10^{-9} - 0.07*3 + \frac{20}{180}\pi)\hat{y}$$

$$H_s = 1.94\hat{x} - 2.1\hat{y} \rightarrow H_x = 1.94 \; A/m$$

(c)

$$H_s = 2\cos(0 - 0 - \frac{40}{180}\pi)\hat{x} + 3\cos(0 - 0 + \frac{20}{180}\pi)\hat{y} = 1.53\hat{x} - 2.819\hat{y}$$

$$|H_s| = 3.2 \; A/m$$